All Work and No Play

Auburn Mountainview

Karl Steffin, 2006

8/7/2024

Work and Energy

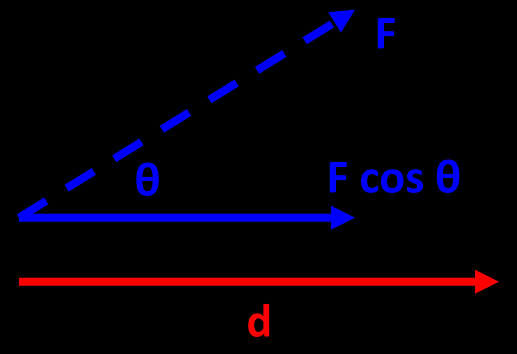
- Using terms like work and energy are common in everyday life.
  - These words are formally used and defined in physics.
- When thinking of work normally two main ideas are considered:
  - How an object is moved (push, pull, …) these are already known as Forces.
  - How far an object is moved (distance).

Work

- Work is defined as the force applied to an object over a distance.

$$W = F \bullet d \quad ([s]\ N \bullet m \rightarrow \text{Joule or J})$$

- Many times, the force applied is not in the direction of the movement of the object. (some of the Force is wasted)
- In these cases, $W = F \bullet \cos\theta \bullet d$



Example I

A cable lifts a 1,200.00-kg elevator at a constant velocity for a distance of 35.00 meters. What is the work done?

Since the force and the distance are in the same direction the basic formula can be used.

The velocity is constant, so the only forces are the Tension force of the cable which is equal and opposite to the weight.

Example I: Solved

$$W = F_T \cdot d$$

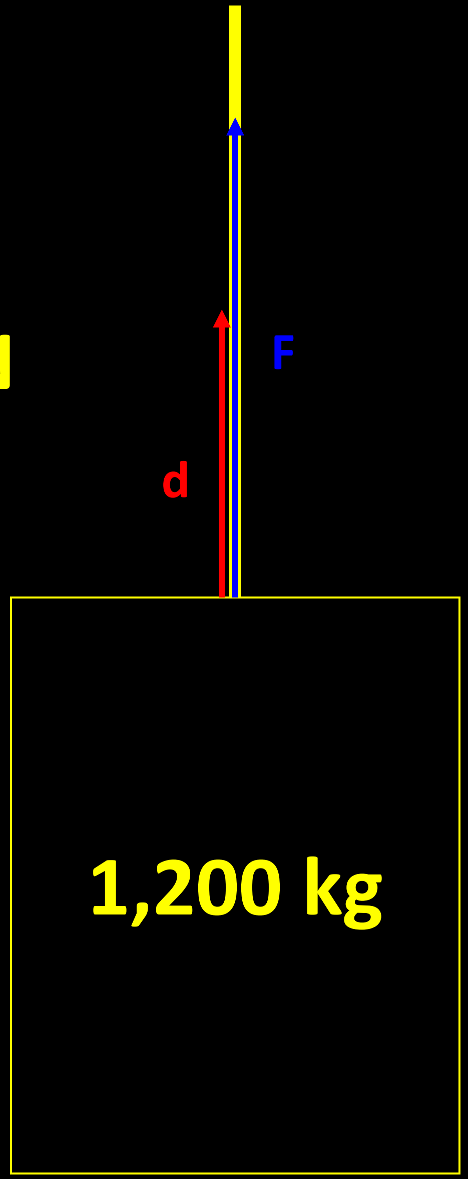
$$W = 11760 - N \cdot 35 - m$$

$$W = 411600 - J$$

F_E= -11760-N

F_T= 11760-N

d= 35-m



$$W = 4.12x10^5 - J$$

Example II

A weightlifter benches 760.00 - N up and then down once at a constant velocity. Assuming his arm moves through .65 meters each way, what is the work done both up and down?

The force applied by the lifter is always up so the basic formula works for the up press but not the down.

Example II: Solved

F= 760-N

d= .65-m

Up

$$W = F \cdot d$$

$$W = 760 - N \cdot .65 - m$$

$$W = 49400 - J$$

$$W = 4.94x10^4 - J$$

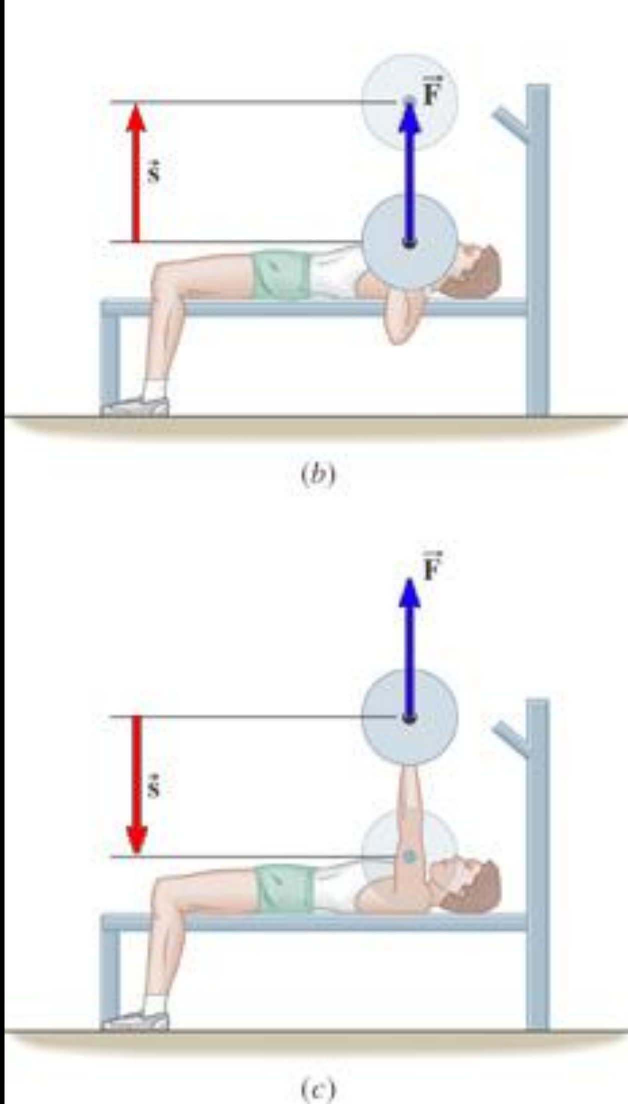
Down

$$W = F \cdot cos\theta \cdot d$$

$$W = 760 - N \cdot cos(180) \cdot .65 - m$$

$$W = -49400 - J$$

$$W = -4.94x10^4 - J$$



Example III

A sailor pulls his boat along a 30.00 meter dock using a rope at a 25.00° angle to the boats movement. How much work is done if he exerts a force of 255.00 Newtons?

Once again, since the force is not in the same direction the expanded formula is used.

Example III: Solved

$$W = F \cdot cos\theta \cdot d$$

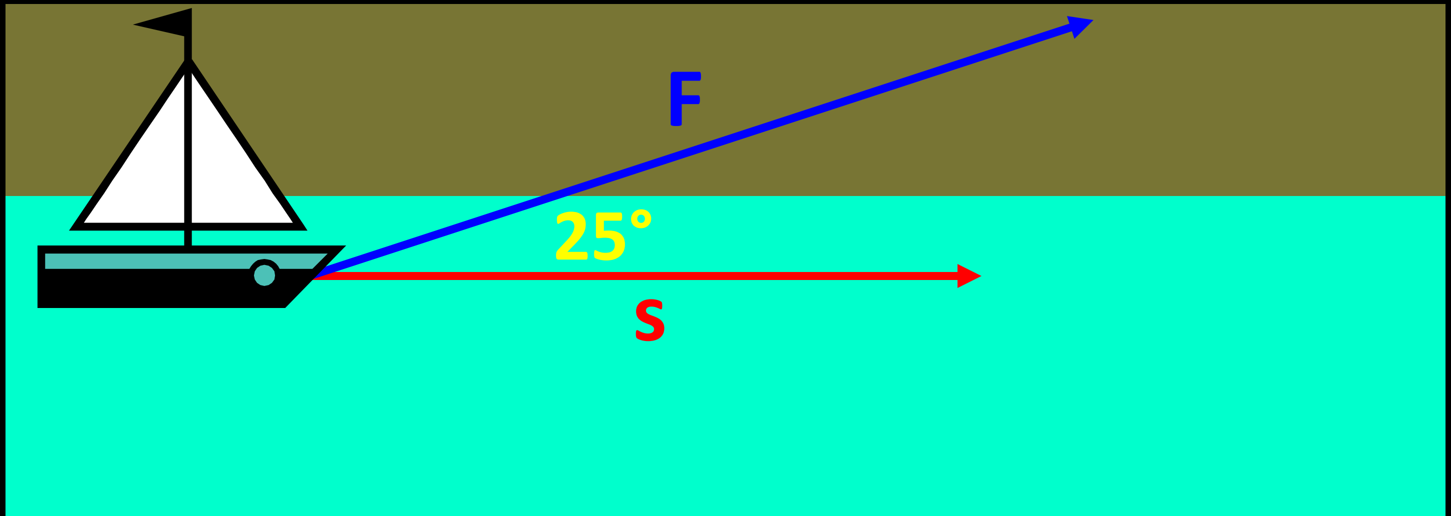
$$W = 255 - N \cdot cos(25) \cdot 30 - m$$

$$W = 6933.25 - J$$

F_T= 255-N

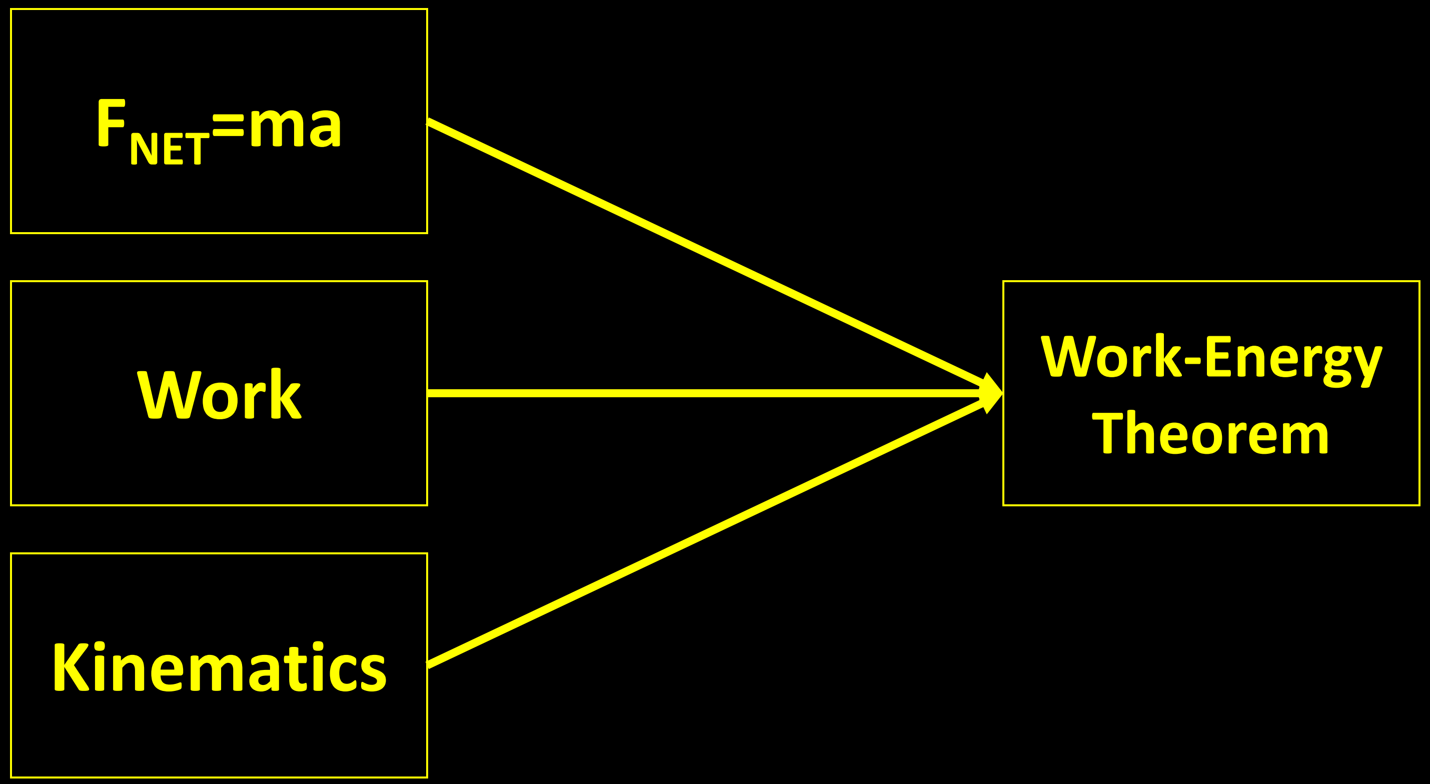
d= 30-m

Θ= 25°



Relationships

- **Just like Impulse and Momentum are related, Work and Energy can also be combined:**



Work-Energy Derived

$$W = F \cdot d$$

$$W = ma \cdot d$$

$$W = m \cdot \frac{\left(v_f^2 - v_o^2\right)}{2}$$

$$F = ma$$

$$v_f^2 = v_o^2 + 2ad$$

$$\frac{\left(v_f^2 - v_o^2\right)}{2} = ad$$

$$W = \frac{1}{2} m \cdot v_f^2 - \frac{1}{2} m \cdot v_o^2$$

Moving (Kinetic) Energy is defined as: KE = ½ mv^2 ([s] also J)

So….

$$W = KE_f - KE_o$$

$$W = \Delta KE$$

Example IV

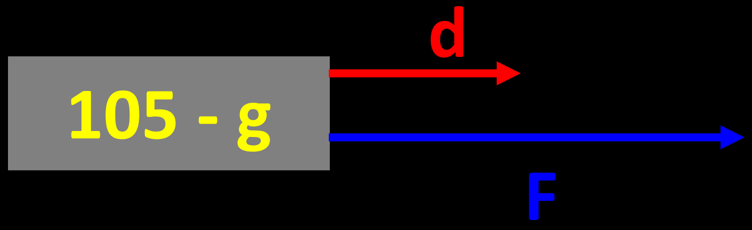
A 105.00-g hockey puck is sliding across the ice. Jordan Eberle exerts a 4.50-N force over a distance of .15-m.

a) How much work does Jordan do to the puck?

b) How much Energy is transferred to the puck?

c) Assuming the initial velocity of the puck is 5.00-m/s, what is the final velocity of the puck?

Example IV: Solved



m= .105-kg

F= 4.5-N

d= .15-m

a) $W = F \cdot d$

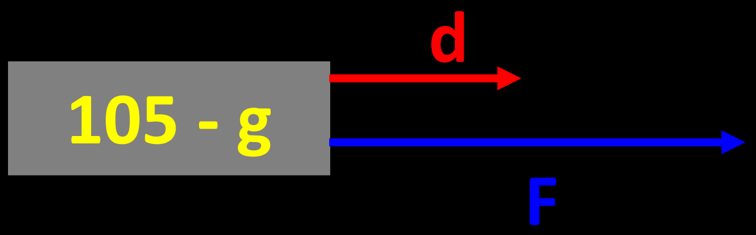
$W = 4.5 - N \cdot .15 - m$

$$W = 6.75x10^{-1} - J$$

b) $W = \Delta KE$

$$\Delta KE = 6.75x10^{-1} - J$$

Example IV: Solved



m= .105-kg

F= 4.5-N

d= .15-m

c) $\Delta KE = \frac{1}{2}m(v_f^2 - v_o^2)$

$$.675 - J = \frac{1}{2} \cdot .105 - kg(v_f^2 - (5 - \frac{m}{s})^2)$$

$$12.875 - \frac{m^2}{s^2} = v_f^2 - 25 - \frac{m^2}{s^2}$$

$$37.857 - \frac{m^2}{s^2} = v_f^2$$

$$\boxed{v_f = 6.15 - \frac{m}{s}}$$

Power

- It can be useful to know the rate of time is takes to produce an amount of Energy.
- This rate is called Power and defined by:

$$P = \frac{\Delta W}{\Delta t}$$ [s] (SI: J/s → Watt, W)

- Due to the magnitude of power produced traditionally kilo or Mega Watts are used.
  - In the case of DeLoreans… 1.21 GW may be needed.

Example V

An electric motor is used to lift an elevator 9.00 meters in 15.00 seconds by exerting an upward force of 1.20 x 10^4 -N. How much power does the motor produce in both Watts and kiloWatts?

Example V: Solved

$$W = F \cdot d$$

$$W = 1.2x10^4 - N \cdot 9 - m$$

$$W = 1.08x10^5 - J$$

$$P = \frac{W}{t}$$

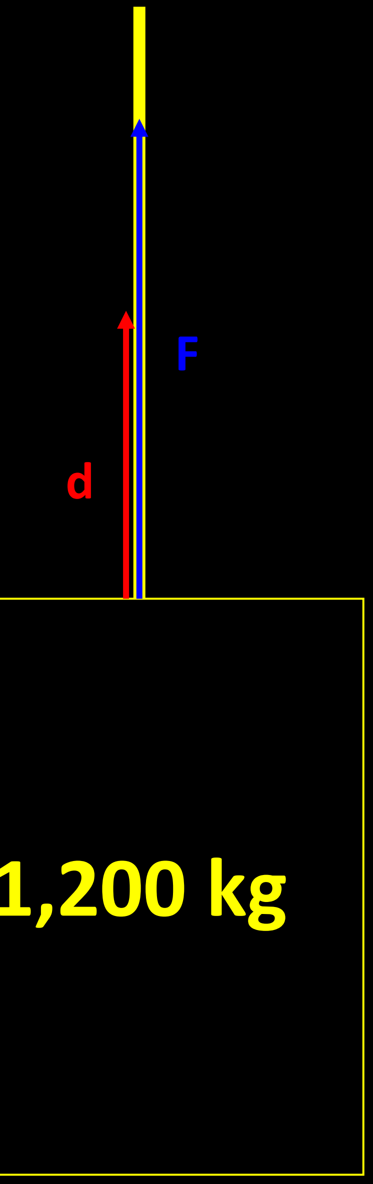
$$P = \frac{1.08x10^5 - J}{15 - s}$$

F_E= -11760-N

F_T= 11760-N

d= 9-m

t= 15-s



$$P = 7200.00 - W$$

$$P = 7.20 - kW$$

Machines

- Machines: Devices that transform the direction and/or magnitude of force.
  - There are six simple machines.
  - To do work an effort force must applied over a distance.
  - The machine translates this to a new resistance force and distance.
- The ratio of resistance force to effort force is called the Mechanical Advantage.

$$MA = \frac{F_r}{F_e}$$

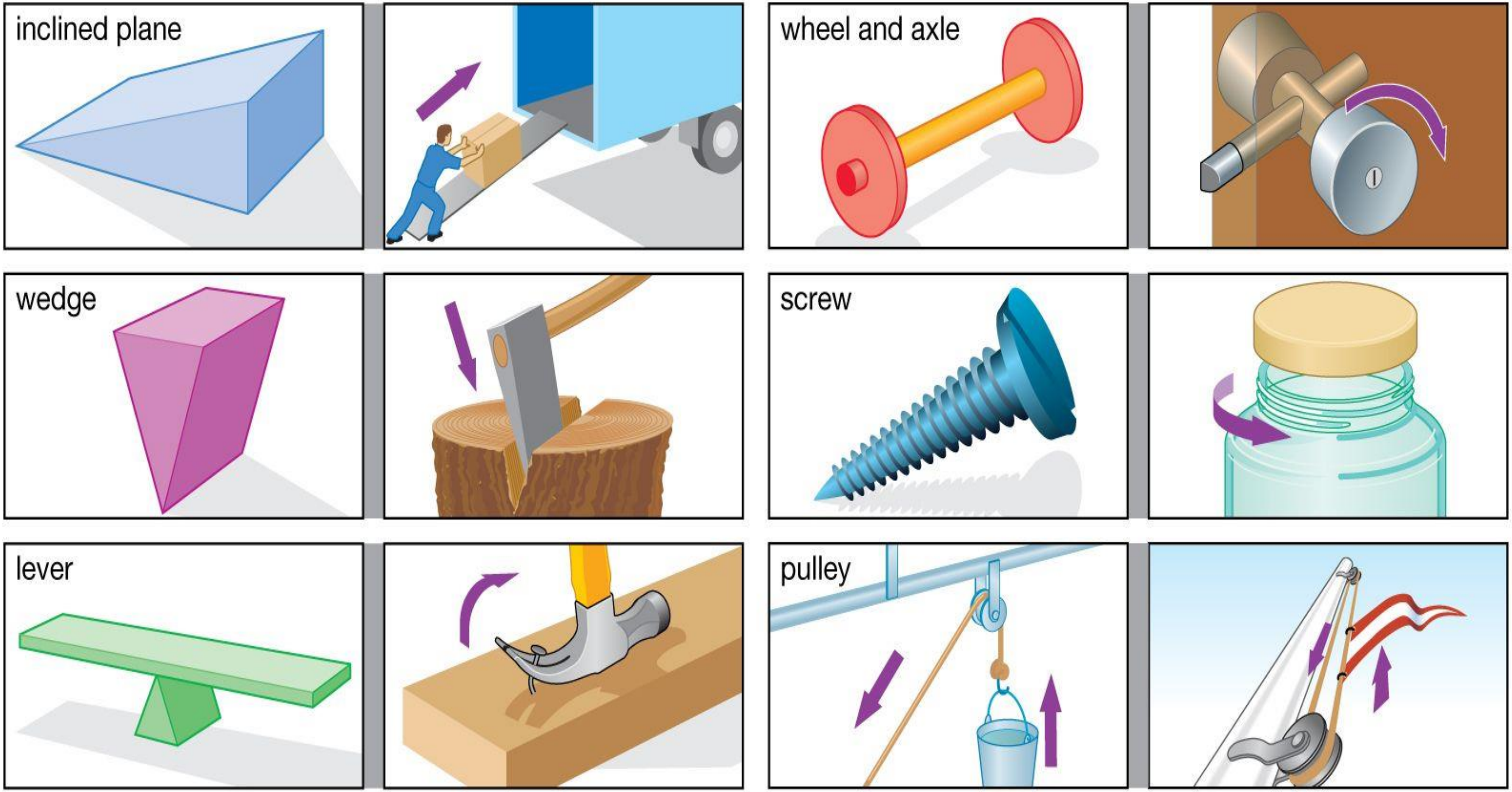
Something from Nothing

- Using a machine is about multiplying forces or distances to an advantage.
- A perfect system (all energy conserved) is defined by an Ideal Mechanical Advantage.

$$IMA = \frac{d_e}{d_r}$$

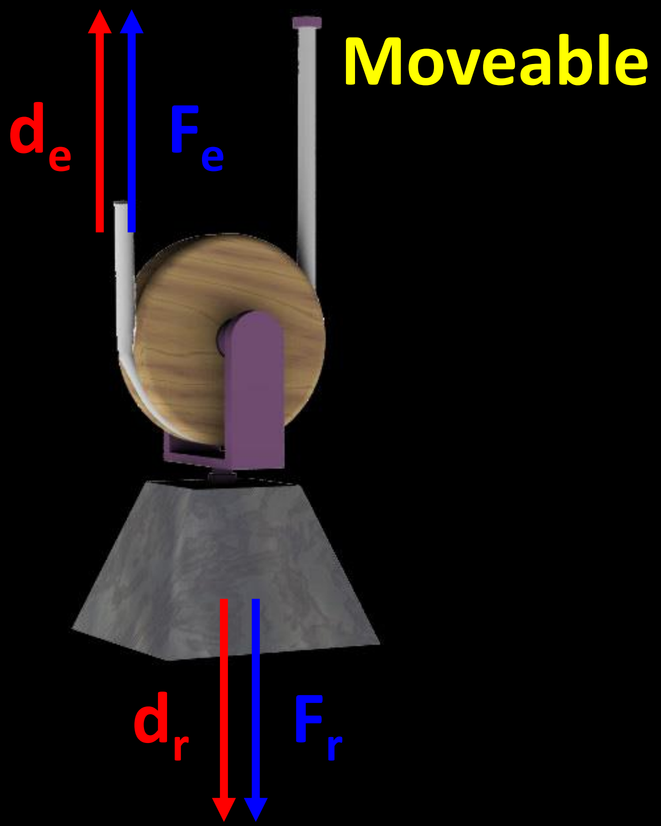
- With these two formulas the efficiency of the machine can be calculated.

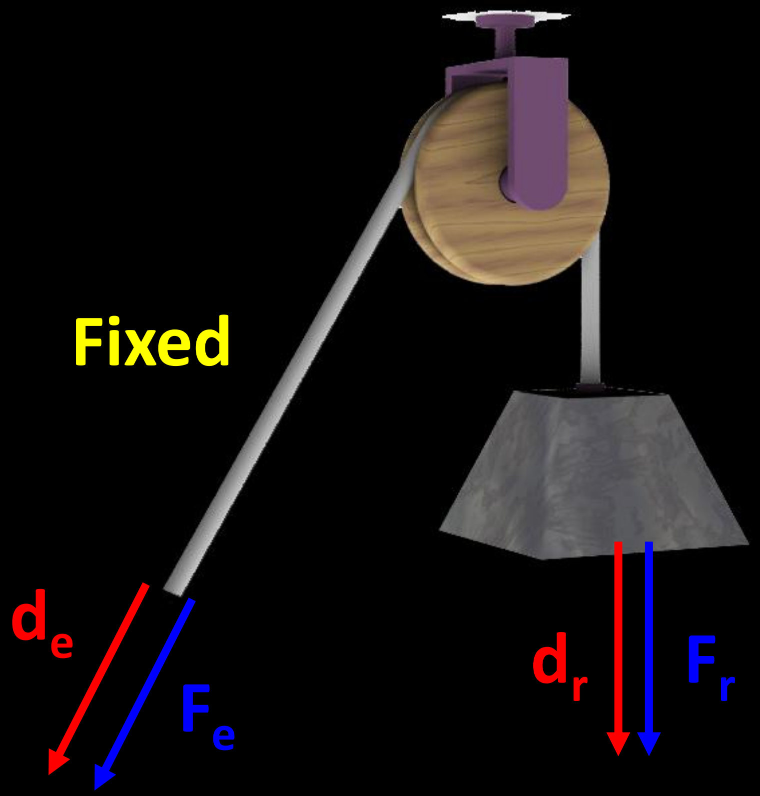
$$Eff = \frac{MA}{IMA} = \frac{F_r d_r}{F_e d_e} = \frac{W_{out}}{W_{in}}$$

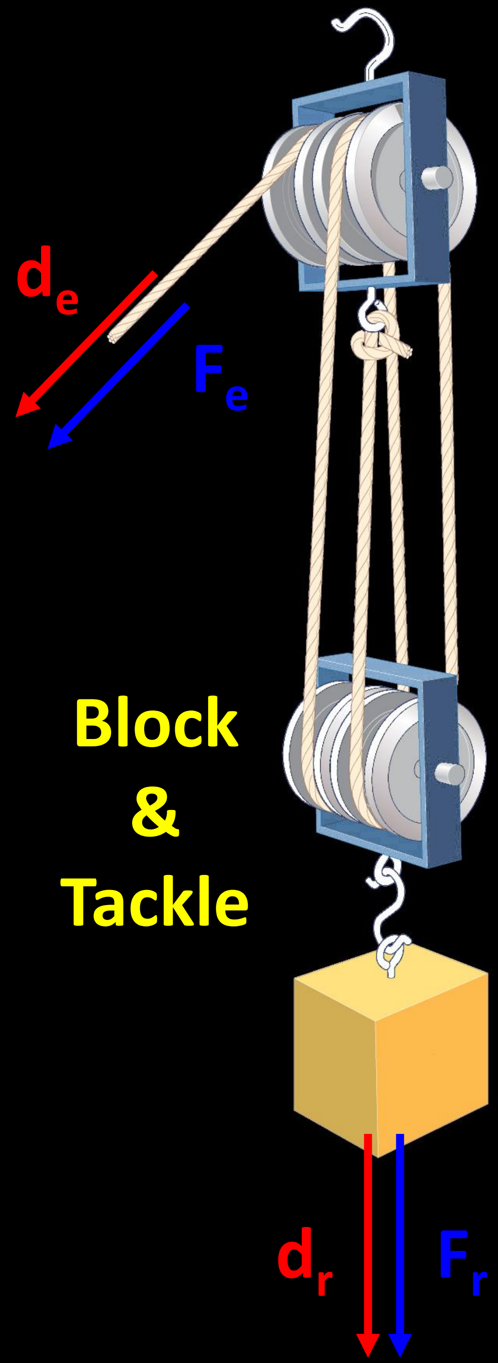

inclined plane
wheel and axle
wedge
screw
lever
pulley

Simple Machines: Pulley

- Fr/Gr: little pivot (ca 1900 BCE)
  - A wheel with a groove along its edge.

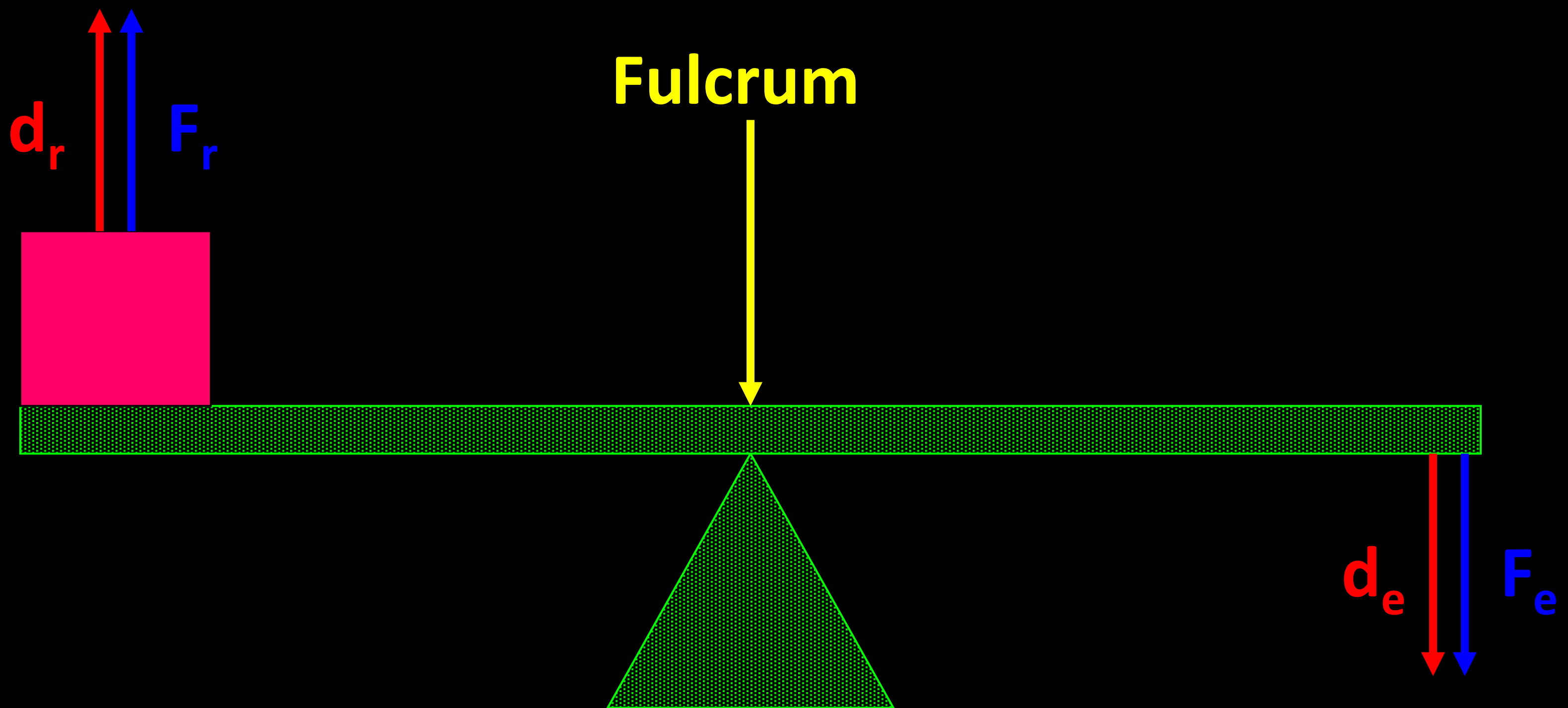






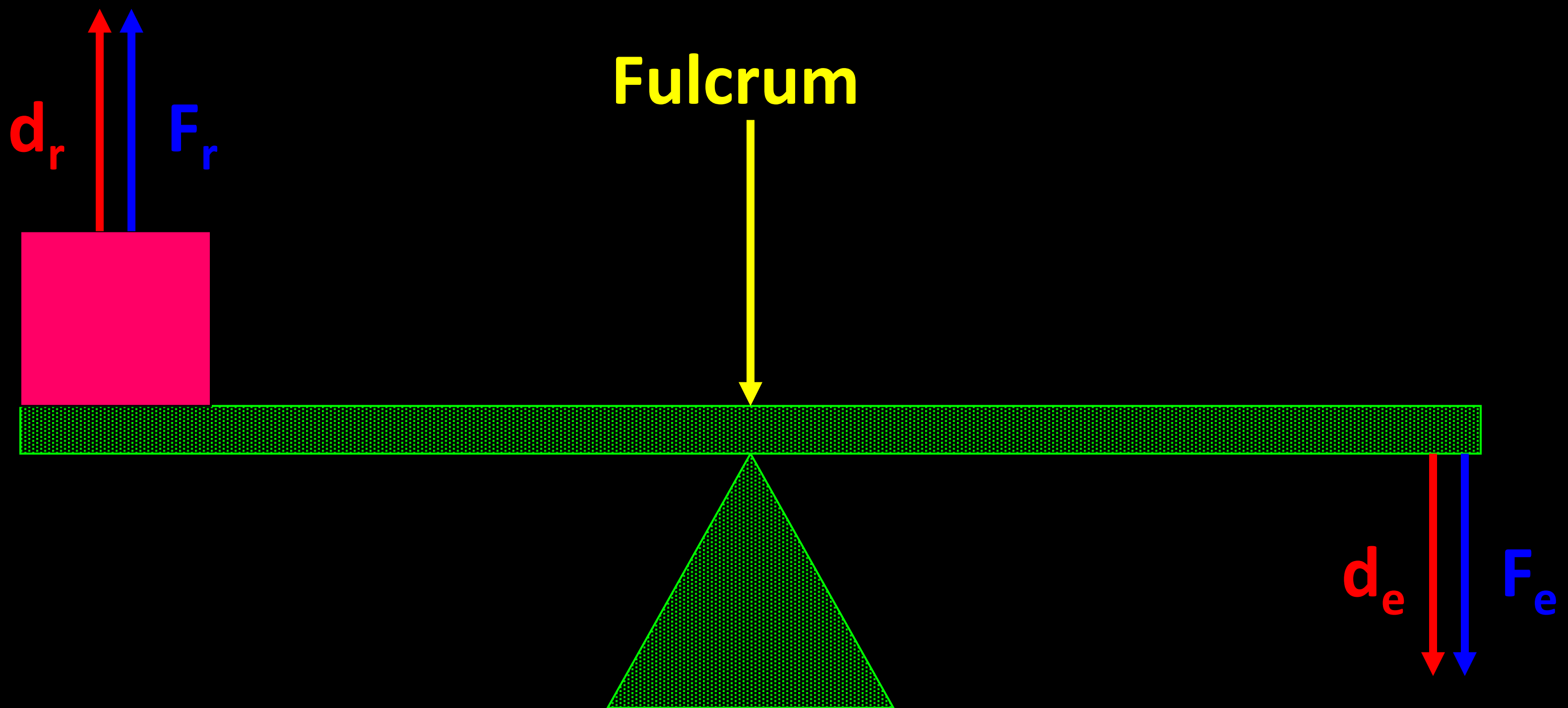
Simple Machines: Lever

- Fr: to raise (ca 5000 BCE)
  - A rigid object that is used with an appropriate fulcrum (pivot) point. There are three classes.



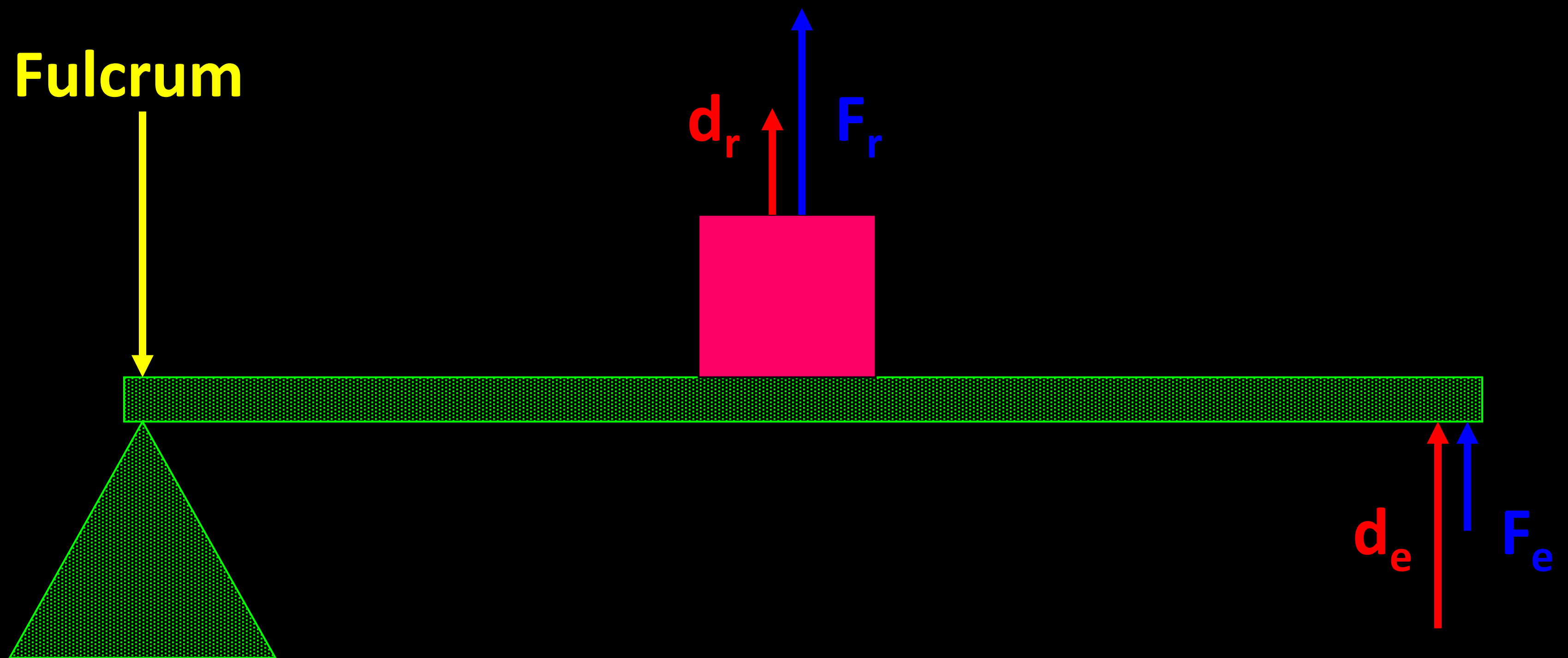
Simple Machines: 1st Class Lever

- Fulcrum located in between the resistive force and effort force.
  - Ex: seesaw, crowbar, scissors, trebuchet



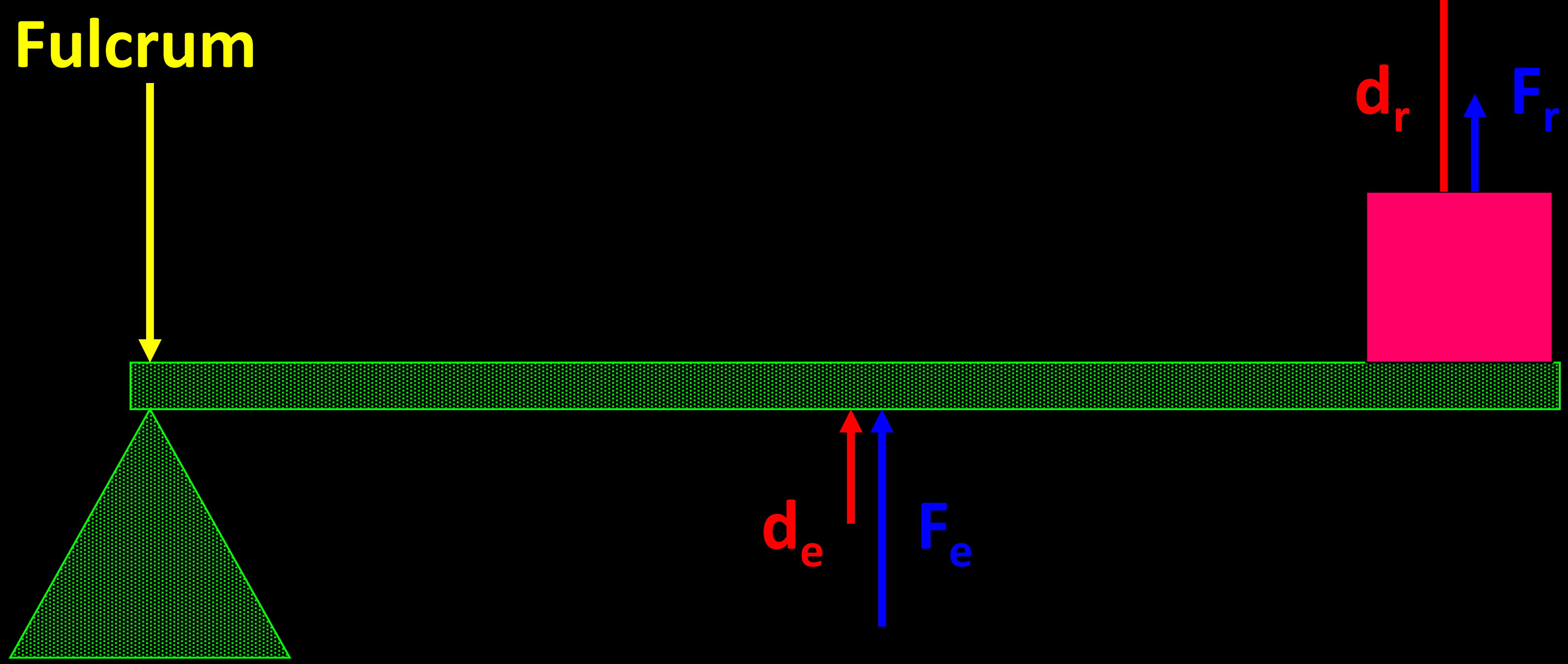
Simple Machines: 2nd Class Lever

- 2nd Class: Resistive force located in between effort force and the fulcrum.
  - Ex: door, crowbar, wheelbarrow, oars



Simple Machines: 3rd Class Lever

- Effort force located in between resistive force and the fulcrum.
  - Ex: your arm, catapult, baseball bat, shovel



Simple Machines: Levers

- Here is a pneumonic to remember:

FRE-123

What's in the center?

Fulcrum (pivot): 1st class

Resistive (load): 2nd class

Effort (you): 3rd class

Example VI

A piano (155.00-N) is attached to a pulley. Pulling on the rope 12.00-m lifts the piano 5.00-m. The efficiency of the machine is 95%.

1. What is the IMA of the System?
2. What is the MA of the System?
3. What is the effort force?

Example VI

F_r= 155.00-N
F_e=
d_r= 5.00-m
d_e= 12.00-m
Eff = .95

$$IMA = \frac{d_e}{d_r}$$

$$IMA = \frac{12 - m}{5 - m}$$

$$IMA = 2.40$$

$$Eff = \frac{MA}{IMA}$$

$$.95 = \frac{MA}{2.40}$$

$$MA = 2.28$$

$$MA = \frac{F_r}{F_e}$$

$$2.28 = \frac{155 - N}{F_e}$$

$$F_e = 67.98 - N$$

A Few Words About Energy

- Within a closed system energy is always conserved.
  - Conservation of Mass & Energy Law
- Since there are so many types of energy it is always important to find the total energy of a system.
- The two major forms of mechanical energy are Kinetic (moving) and Potential (stored).
- Review that energy is always a scalar (Joules).

Example VII

While passing you on the road, a car (875.00-kg) speeds up from 22.00-m/s to 44.00-m/s. What are the initial and final kinetic energies of the car?

$$KE_o = \frac{1}{2} m \cdot v_o^2$$

$$KE_o = \frac{1}{2} \cdot 875 - kg \cdot (22 - \frac{m}{s})^2$$

$$KE_o = 211750 - J$$

m= 875-kg

v_o= 22-m/s

v_f= 44-m/s

$$KE_o = 2.12x10^5 - J$$

Example VII

$$KE_f = \frac{1}{2} m \cdot v_f^2$$

$$KE_f = \frac{1}{2} \cdot 875 - kg \cdot (44 - \frac{m}{s})^2$$

$$KE_f = 847000 - J$$

m= 875-kg
v_o= 22-m/s
v_f= 44-m/s

$$KE_f = 8.47x10^5 - J$$

Two Types of Stored Energy

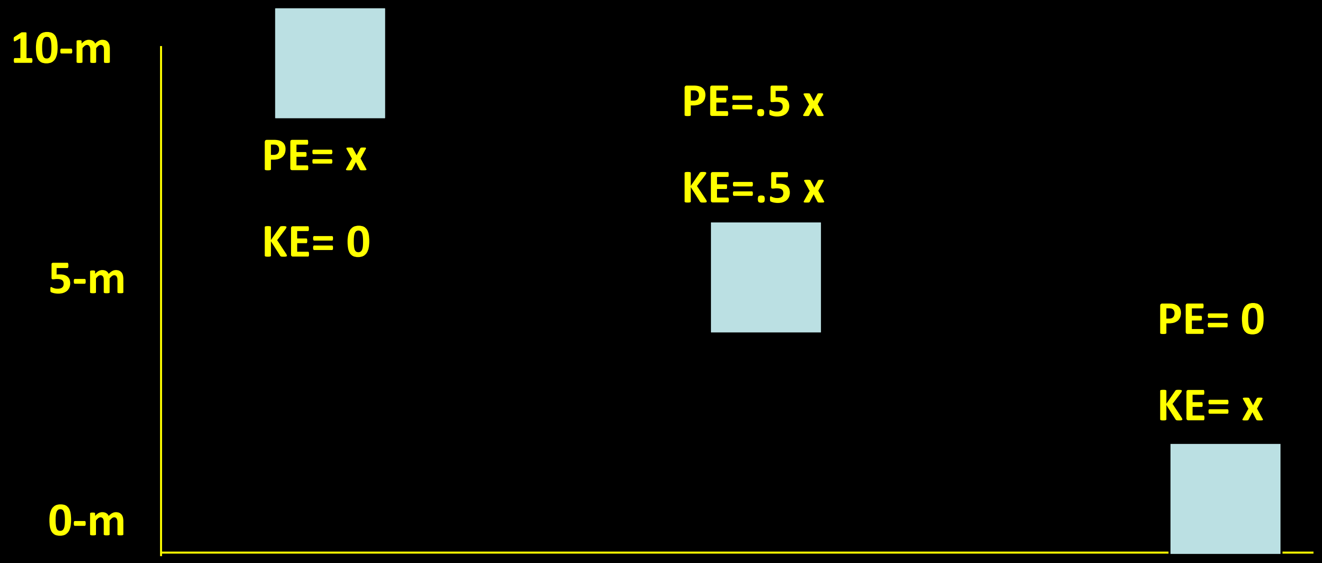
- Potential (Gravitational): If a boulder is sitting on the top of a hill, it has the ability to roll down to the bottom (because of gravity).
  - At the top the boulder has a full ‘hill’ of stored energy.
  - Halfway down it only has half a ‘hill’ of energy.
  - At the bottom it has no more energy from the hill.
- Potential (Elastic): If a spring is compressed or stretched then it has the ability to release energy.

$PE_g = mgh$

$PE_e=1/2kx^2$

The Flow of Energy

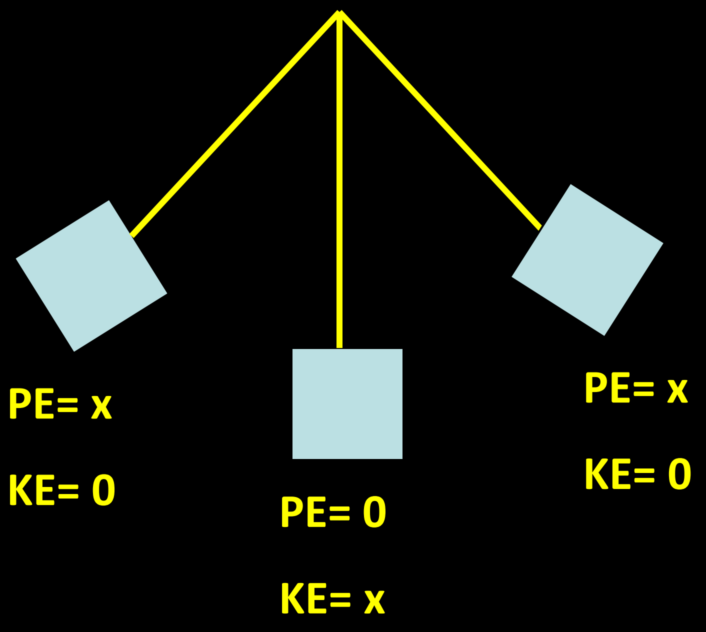
- When looking at closed systems normally there is a flow between Kinetic Energy and Potential Energy. Look at the following:

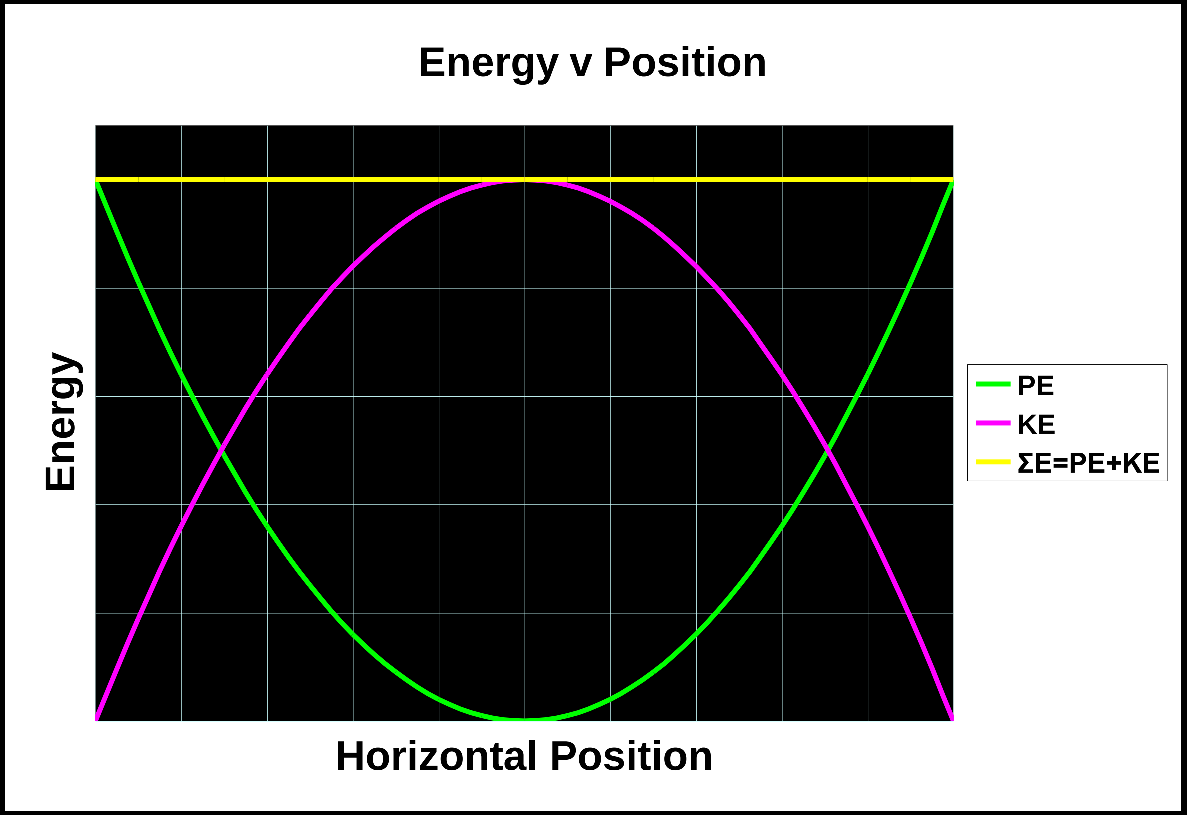


In all cases the total energy is conserved.

Another Exchange

- Here is another look with a graph.





KE-PE Example I

An icicle with mass 15.00-kg falls from a roof 8.00-m above the ground.

1. What is the KE of the icicle before it hits the ground?
2. What is the speed of the icicle before it hits the ground?

$$PE_{Top} = KE_{Bottom}$$

$$m \cdot g \cdot h = KE_{Bottom}$$

$$15 - kg \cdot 9.8 - \frac{m}{s^2} \cdot 8 - m = KE_{Bottom}$$

PE=

m= 15-kg

g= 9.8-m/s^2

h= 8-m

$$KE_{Bottom} = 1176.00 - J$$

KE-PE Example I

An icicle with mass 15.00-kg falls from a roof 8.00-m above the ground.

- What is the speed of the icicle before it hits the ground?

$$KE_B = \frac{1}{2} \cdot m \cdot v^2$$

$$1176 - J = \frac{1}{2} \cdot 15 - kg \cdot v^2$$

$$156.8 - \frac{m^2}{s^2} = v^2$$

KE= 1176-J

m= 15-kg

v=

$$v = 12.52 - \frac{m}{s}$$

KE-PE Example II

A ball slams against the ground at 20.00-m/s. Assuming perfect elasticity how high will it bounce?

- Note even without mass it is possible to solve.
- Another way to look at this $v_{max\ bottom} = h_{max\ top}$.

$$m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot v^2$$

$$9.8 - \frac{m}{s^2} \cdot h = \frac{1}{2} \cdot (20 - \frac{m}{s})^2$$

g= 9.8-m/s^2

h=

v= 20-m/s

$$h = 20.41 - m$$

Collisions

Two cars (m_1= 575.00-kg, m_2= 1575.00-kg) traveling down the road (v_1= 15.00-m/s, v_2= 5.00-m/s) rear end each other and stick together.

1. Pre-Crash what is the total Energy in the system?
2. What is the final velocity of the locked cars?
3. What is the final KE of the cars?

Collisions

1. What is the total Energy in the system?

m$_1$= 575-kg

m$_1$= 1575-kg

v$_1$= 15-m/s

v$_2$= 5-m/s

$$\Sigma E = KE_1 + KE_2$$

$$\Sigma E = \frac{1}{2} \cdot 575 - kg \cdot \left(15 - \frac{m}{s}\right)^2 + \frac{1}{2} \cdot 1575 - kg \cdot (5 - \frac{m}{s})^2$$

$$\Sigma E = 64687.5 - J + 19687.5 - J$$

$$\Sigma E = 84375.00 - J$$

$$\boxed{\Sigma E = 8.43x10^4 - J}$$

Collisions

2. What is the final velocity of the locked cars?

m_1= 575-kg
m_2= 1575-kg
v_1= 15-m/s
v_2= 5-m/s

$$p_o = p_f$$

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_f$$

$$575 - kg \cdot 15 - \frac{m}{s} + 1575 - kg \cdot 5 - \frac{m}{s} = (575 + 1575) - kg \cdot v_f$$

$$16500 - kg\frac{m}{s} = 2150 - kg \cdot v_f$$

$$\boxed{v_f = 7.67 - \frac{m}{s}}$$

Collisions

3. What is the final KE of the cars?

$$KE = \frac{1}{2} \cdot (m_1 + m_2) \cdot v_f^2$$

$$KE = \frac{1}{2} \cdot (2150 - kg) \cdot (7.67 - \frac{m}{s})^2$$

$$KE = 63314 - J$$

m_1= 575-kg
m_1= 1575-kg
v_f= 7.67-m/s

$$KE = 6.33x10^4 - J$$

Energy in the system is lost
(Sound, Heat, Deformation)