# Standing Waves on a String

### INTRODUCTION

When you shake a string, a pulse travels down its length. When it reaches the end, the pulse can be reflected. A series of regularly occurring pulses will generate traveling waves that, after reflection from the other end, will interfere with the oncoming waves. When the conditions are right, the superposition of these waves traveling in opposite directions can give rise to something known as a "standing wave." That is, there appear to be stationary waves on the string with some parts of the string hardly moving at all and other regions where the string experiences a large displacement. In this lab you will investigate the various factors that give rise to this phenomenon.

#### **OBJECTIVES**

In this experiment, you will

- Adjust the frequency of the driver so that the string vibrates in the fundamental mode.
- Set up other standing wave patterns on the string.
- Relate the frequency of the various harmonics to that of the fundamental mode of vibration.
- Describe the terms amplitude, frequency, wavelength, node, and antinode as they relate to vibrating strings.
- Determine the velocity of waves in the string.
- Relate wave velocity to the tension of the string and its linear density.

#### MATERIALS

Vernier LabQuest Logger *Pro* or LabQuest App **or** a computer with the Vernier Power Amplifier computer program Vernier Power Amplifier Vernier Power Amplifier Accessory Speaker Vernier Ultra Pulley and support rod 3–4 m length of rope or a tightly coiled spring ("snaky") mass hanger drilled or slotted lab masses ring stand and right-angle clamp meter stick or metric tape elastic cord 1 m lengths of string with varying thickness

# **PRE-LAB INVESTIGATION**

Stretch the rope between you and a partner, who will serve as the fixed end of the rope. Shake the rope up and down, gradually increasing the frequency until you obtain a pattern like the one shown in Figure 1, below. If a tightly coiled spring is available, you can achieve the same effect by stretching it on a smooth floor between you and your partner and shaking it back and forth until you obtain this pattern:



Figure 1

This is the simplest standing wave one can set up on a string or spring held at both ends. It is known as the fundamental frequency of vibration (also called the first harmonic,  $f_1$ ). It is the primary mode of vibration of a guitar string when it is plucked in the middle.

Note that the displacement of the string (or spring) is greatest in the middle and least at the ends where it is held fixed. In your class discussion you should record definitions for the terms *nodes*, *antinodes*, and *amplitude*. Your instructor may show you an animation illustrating how the interference of traveling waves can give rise to standing waves.

Now return to the rope or spring. Increase the rate at which you shake it until you obtain a waveform with a node in the middle as well as at either end. This waveform is often called the second harmonic,  $f_2$ . At this rate of shaking (frequency) you are producing a complete wave. The distance between the two fixed ends is the wavelength,  $\lambda$ , of the standing wave. How many antinodes appear? What fraction of a complete wave is represented by the fundamental?

# PART 1 SETTING UP STANDING WAVE PATTERNS PROCEDURE

1. Set up the apparatus as shown in Figure 2. Attach the hook that comes with the Power Amplifier Accessory Speaker to the speaker. Tie one end of the elastic cord to the hook and the other end to a support rod. The elastic cord should be stretched just tightly enough that it does not sag visibly, but is not too taut (the length of the cord should be 60–80 cm). Record the length of the cord. Turn on the power to the Power Amplifier.



Figure 2 Power Amplifier and Accessory Speaker

2. Connect the speaker leads to the Power Amplifier, then choose one of the following options to drive the speaker.

LabQuest with Logger Pro

- a. Use the mini stereo cable that came with the amplifier to connect the Speaker Out port on LabQuest and the Audio In port on the amplifier.
- b. In Logger *Pro*, choose Set Up Sensors ► Show All Interfaces from the Experiment menu.
- c. Click Power Amplifier. Select Sine as the Waveform.
- d. The default value of 2.0 V is suitable. Change the initial frequency to about 20 Hz. Use the up and down arrows to adjust the frequency or use the parameter box to enter the desired value.
- e. Click Start.

LabQuest as a standalone device

- a. Use the mini stereo cable that came with the amplifier to connect the Speaker Out port on LabQuest and the Audio In port on the amplifier.
- b. Launch Power Amplifier from the LabQuest Home menu. The default setting of 2.0  $V_{AC}$  is suitable. Change the frequency to about 20 Hz.
- c. Tap start. Use the up and down arrows to adjust the frequency or enter the value in the parameter field.

Power Amplifier computer program and the computer's audio output.

- a. Use the mini stereo cable that came with the Power Amplifier to connect the speaker out port on your computer and the Audio In port on the amplifier.
- b. Set the computer's sound output on and at maximum volume.
- c. Start the Vernier Power Amplifier computer program.
- d. The default value of 2.0 V is suitable. Change the initial frequency to about 20 Hz. To adjust the frequency, use the up and down arrows or use the frequency control box to enter the desired value.
- e. Click Start.
- 3. Adjust the frequency until you have generated the fundamental mode of vibration (greatest amplitude in the middle and nodes only at the ends) on the cord. Note: If you need to stop the vibration of the string, disconnect a lead from the amplifier to the speaker rather than turning off the amplifier. As you approach the optimal frequency, make small adjustments and wait a few seconds after each adjustment to allow the system to stabilize. To ensure that you have reached the optimal frequency, increase the frequency gradually until the amplitude begins to decrease, then reduce it until the amplitude again appears to have reached its maximum. Record this frequency as  $f_1$  in your data table.
- 4. Without changing the length of the cord, increase the driving frequency gradually until the second harmonic,  $f_2$ , appears. Record this frequency in your data table. Note how the amplitude of the antinodes compares to that in the fundamental. Sketch the waveform that you observe between the two fixed points at the ends of the cord.
- 5. Use the values of  $f_1$  and  $f_2$  to predict the value of  $f_3$ . Set the driving frequency to this value, then adjust the frequency until the amplitude of the standing wave is at a maximum. Record this frequency in your data table. Note where the nodes appear on the cord. Sketch the waveform that you observe; be sure to keep the distance between the ends of the strings unchanged. How many waves are visible?

6. Continue this process until you have generated the fifth harmonic,  $f_5$  on the cord. Stop driving the speaker and turn off the power amplifier.

# **EVALUATION OF DATA**

- 1. Compare the frequencies of the higher harmonics  $(f_2, f_3, \text{etc.})$  to that of the fundamental,  $f_1$ . Write a statement that describes the relationship you find.
- 2. For each of your standing waveforms, determine the wavelength,  $\lambda$ , (in m) of the standing wave.
- 3. Choose New from the File menu. Manually enter values so as to produce a graph of wavelength *vs*. frequency. Instead of Hz, use 1/s as the units for frequency. Write a statement that describes the relationship between the wavelength and frequency.
- 4. If your graph of wavelength *vs.* frequency is not linear, take steps to modify a column so as to produce a linear relationship. When you have done so, save your file and (if possible) print a copy of your original and linearized graphs.
- 5. Record the equation of the line that best fits your linearized graph. Examine the units of the slope of the line. Considering that a standing wave on a string results from the interference of waves traveling back and forth, what physical property of the system is suggested by the units of the slope?
- 6. Rearrange the equation so as to express frequency and wavelength of the standing waves in terms of this variable.

# PART 2 FACTORS AFFECTING THE SPEED OF WAVES ON A STRING PROCEDURE

- 1. Replace the elastic cord used in Part 1 with a less stretchy braided string. Find the mass and length of your string to calculate the linear density in kg/m; record this value in your data table.
- 2. Tie one end to the hook on the speaker and the other end to a very light mass hanger. Suspend the string over a pulley, as shown in Figure 3. Use the lightest masses you have available to keep the string from sagging. Measure the length of the string between the hook and the point of contact with the pulley.



Figure 3

- 3. As you did in Part 1, adjust the frequency of the driver so as to set up the second harmonic,  $f_2$ , (one complete wave) in the string. Record this frequency.
- 4. Increase the mass on the hanger so as to approximately double the tension in the string. Your instructor may advise you about how to appropriately increment the mass.
- 5. Increase the frequency until the second harmonic again appears. As you did in Part 1, take steps to ensure that you have reached the optimal frequency. Record this value as well as the tension provided by the hanging mass.
- 6. Continue adding mass to the hanger to increase the tension on the string and adjusting the frequency to produce the second harmonic (one complete wave) until you have seven or eight data pairs. When you are done, turn off the driver.
- 7. If time permits,<sup>1</sup> repeat Part 2, Steps 1–3 with different types of string, but use an intermediate value of the tension on the string. Record the value of the frequency required to produce the second harmonic on the string.

# **EVALUATION OF DATA**

- 1. From your understanding of the relationship of velocity, frequency, and wavelength of waves, determine the velocity of each of the standing waves you generated for the first string.
- 2. Choose New from the File menu. Enter your data manually to produce a graph of velocity *vs.* tension. Write a statement that describes the relationship between the velocity and the tension in the string.
- 3. If your graph of velocity *vs.* tension is not linear, create a new calculated column to adjust the velocity so as to produce a linear relationship. When you have done so, save your file and (if possible) print a copy of your original and then linearized graph.
- 4. Record the equation of the line that best fits your linearized graph. Simplify the units of the slope, and then compare these to the units of linear density.
- 5. Predict the relationship between the quantity on the vertical axis of your linearized graph and the linear density.
- 6. To test this prediction, determine the velocity of the standing waves in each of the different types of string for which you have data.
- 7. Choose New from the File menu. Enter your data manually to produce a graph of velocity *vs.* linear density.
- 8. If your graph of velocity *vs.* linear density is not linear, take the necessary steps to adjust the velocity and/or linear density so as to produce a linear relationship. When you have done so, save your file and (if possible) print a copy of your original and then linearized graph.
- 9. Suggest a general equation relating wave velocity, tension, and linear density of the medium.

<sup>&</sup>lt;sup>1</sup> If not, obtain values of frequency, wavelength, and tension from other groups that used different types of string.

## **EXTENSION**

The behavior of a transverse wave on a string can be modeled by viewing the string as a collection of linked particles capable of undergoing up-and-down oscillations. A simulation of this model can be found at

http://phet.colorado.edu/sims/wave-on-a-string/wave-on-a-string\_en.html

Observe the behavior of the string when you provide pulses, and then cause the end to oscillate. Speculate as to why the tension of the string affects the wave speed. Change the tension in the simulations to test your ideas.