All Work and No Play

Auburn Mountainview
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Work and Energy

- Using terms like work and energy are common in every day life.
  - These words are formally used and defined in physics.
- When thinking of work normally two main ideas are considered:
  - How an object is moved (push, pull, ...) these are already known as Forces.
  - How far an object is moved (distance).
Work

• Work is defined as the force applied to an object over a distance.
  \[ W = F \cdot d \ ( [s] \text{N} \cdot \text{m} \rightarrow \text{Joule or J}) \]

• Many times the force applied is not in the direction of the movement of the object.

• In these cases the expanded formula \( W = F \cdot \cos \theta \cdot d \) should be used.
Example I

- A cable lifts a 1,200.00-kg elevator at a constant velocity for a distance of 35.00 meters. What is the work done?
  - Since the force and the distance are in the same direction the basic formula can be used.
  - The velocity is constant so the only forces are the Tension force of the cable which is equal and opposite to the weight.
Example I: Solved

- \( F_T = -F_E = m\cdot g \)
- \( F_T = -F_E = -(1,200\text{-kg} \times -9.8\text{-m/s}^2) \)
- \( F_T = 11,760\text{ - N} \)
- \( W = F\cdot d \)
- \( W = 11,760\text{ - N} \times 35\text{ - m} \)
- \( W = 411,600\text{ - J} \)
- \( W = 4.12 \times 10^5\text{ - J} \)
Example II

- A weight lifter benches 760.00 N up and then down once at a constant velocity. Assuming that his arm moves through .65 meters each way, what is the work done both up and down?

- The force applied by the lifter is always up so the basic formula works for the up press but not the down.
Example II: Solved

a. \( W = F \cdot d \)
\[
W = 760 - N \times 0.65 - m
\]
\[
W = 494.00 - J
\]

b. \( W = F \cos \theta \cdot d \)
\[
W = 760 \, N \cos (180) \times 0.65 \, m
\]
\[
W = -494.00 - J
\]
Example III

• A sailor pulls his boat along a 30.00 meter dock using a rope at a 25.00° angle to the boats movement. How much work is done if he exerts a force of 255.00 Newtons?

• Once again, since the force is not in the same direction the expanded formula is used.
Example III: Solved

- \( W = F \cos \theta \cdot d \)
- \( W = 255 \text{ N} \cos (25^\circ) \times 30 \text{ m} \)
- \( W = 6933.25 \text{ J} \)
A relationship between Impulse and Momentum has been shown, here is another:

\[ F_{\text{NET}} = ma \]

- Work
- Kinematics

Work-Energy Theorem
Work-Energy Derived

\[ W = F \cdot d \]

\[ F \cdot d = m \cdot a \cdot d \]

\[ F \cdot d = m \cdot \left( V_f^2 - V_o^2 \right)/2 \]

If the remaining \( F \cdot d \) is due to an external force then...

\[ KE = \frac{1}{2} m \cdot V^2 \]

Therefore:

\[ W = KE_f - KE_o \]

\[ W = \Delta KE \]
Example IV

A 105.00-g hockey puck is sliding across the ice. A player exerts a constant 4.50-N force over a distance of .15-m.

a) How much work does the player do to the puck?
b) How much Energy is transferred to the puck?
c) Assuming the initial velocity of the puck is 5.00-m/s, what is the final velocity of the puck?
Example IV: Solved

a. \( W = F \cdot d \)
\[
W = 4.5 - N \cdot 0.15 - m
\]
\[
W = 0.675 - J \ldots 6.75 \times 10^{-1} - J
\]

b. \( W = \Delta KE \)
\[
\Delta KE = 6.75 \times 10^{-1} - J
\]

c. \( \Delta KE = \frac{1}{2} m(v_f^2 - v_o^2) \)
\[
6.75 \times 10^{-1} - J = \frac{1}{2} \cdot 0.105 - \text{kg} \cdot (v_f^2 - (5 - \text{m/s})^2)
\]
\[
12.857 - \text{m}^2/\text{s}^2 = v_f^2 - 25 - \text{m}^2/\text{s}^2
\]
\[
v_f^2 = 37.857 - \text{m}^2/\text{s}^2
\]
\[
v_f = 6.15 - \text{m/s}
\]
Power

• Much of physics deals with rates \((v, a)\)
• Another useful idea is the rate of time is takes to produce an amount of Energy.
• This rate is called Power and defined by:
  \[ P = \frac{\Delta W}{\Delta t} \text{ [s]} \] (SI: J/s \(\rightarrow\) Watt, W)
• Due to the magnitude of power produced traditionally kilo or Mega Watts are used.
  – In the case of DeLoreans...
  1.21 GW may be needed.
Example V

- An electric motor is used to lift an elevator 9.00 meters in 15.00 seconds by exerting an upward force of $1.20 \times 10^4$ N. How much power does the motor produce in both Watts and kiloWatts?
Example V: Solved

a. \[W = F \cdot d\]
\[W = 1.2 \times 10^4 \cdot N \cdot 9 \cdot m\]
\[W = 1.08 \times 10^5 \cdot J\]

\[P = \frac{W}{t}\]
\[P = \frac{1.08 \times 10^5 \cdot J}{15.0 \cdot s}\]
\[P = 7,200.00 \cdot W\]
\[P = 7.20 \cdot kW\]
Machines

- Machines: Devices that transform the direction and/or magnitude of force.
  - There are six simple machines.
  - To do work an effort force must applied over a distance.
  - The machine translates this to an new resistance force and distance.

- The ratio of resistance force to effort force is called the Mechanical Advantage.
  \[
  MA = \frac{F_r}{F_e}
  \]
Something from Nothing

- Using a machine is about multiplying forces or distances to an advantage.
- In a perfect system all energy will be conserved (not really possible).
- This perfect system is defined by an Ideal Mechanical Advantage $\rightarrow$ IMA = $d_e/d_r$.
- With these two formulas the efficiency of the machine can be calculated.

Efficiency = $MA/IMA$ or $F_r d_r / F_e d_e$ or $W_{out}/W_{in}$
Simple Machines:

- **Pulley**: A wheel with a groove along its edge. Multiple pulleys: Block and Tackle.
Simple Machines:

- Lever: (Fr. ‘to raise’) a rigid object that is used with an appropriate fulcrum or pivot point. There are three classes of levers.
Simple Machines: Levers

- 1\textsuperscript{st} Class: Fulcrum located in between the resistive force and effort force.
  - Ex: seesaw, crowbar, scissors, trebuchet
Simple Machines: Levers

- 2\textsuperscript{nd} Class: Resistive force located in between effort force and the fulcrum.
  - Ex: door, crowbar, wheelbarrow, oars
Simple Machines: Levers

- 3\textsuperscript{rd} Class: Effort force located in between resistive force and the fulcrum.
  - Ex: your arm, catapult, baseball bat, shovel
Simple Machines: Levers

• Here is a pneumonic to remember:
  FRE-123
  What’s in the center?
  Fulcrum (pivot): 1st class
  Resistive (load): 2nd class
  Effort (you): 3rd class
Example VI

- A piano (155.00-N) is attached to a pulley. Pulling on the rope 12.00-m lifts the piano 5.00-m. The efficiency of the machine is 95%.
  - What is the IMA of the System?
  - What is the MA of the System?
  - What is the effort force?
Example VI

What is the IMA of the System?

\[ \text{IMA} = \frac{d_e}{d_r} \]
\[ \text{IMA} = \frac{12.00\text{-m}}{5.00\text{-m}} \]
\[ \text{IMA} = 2.40 \]

What is the MA of the System?

\[ \text{Eff} = \frac{\text{MA}}{\text{IMA}} \]
\[ 0.95 = \frac{\text{MA}}{2.40} \]
\[ \text{MA} = 2.28 \]
Example VI

• What is the effort Force?

\[
MA = \frac{F_r}{F_e} \\
2.28 = \frac{155.00-N}{F_e} \\
F_e = 67.98-N
\]
A Few Words About Energy

• Within a closed system energy is always conserved.
  – Conservation of Mass & Energy

• Since there are so many types of energy it is always important to find the total energy of a system.

• The two major forms of mechanical Energy are Kinetic (moving) and Potential (stored).
Example VII

• While passing a car on the road, a 875.00-kg car speeds up from 22.00-m/s to 44.00-m/s. What are the initial and final kinetic energies?

Draw a picture and write what you know.

Set up and Solve.

\[ KE_o = \frac{1}{2}mv_o^2 \]
\[ KE_f = \frac{1}{2}mv_f^2 \]

\[ KE_o = \frac{1}{2} \times 875\text{-kg} \times (22.0\text{-m/s})^2 \]
\[ KE_f = \frac{1}{2} \times 875\text{-kg} \times (44.0\text{-m/s})^2 \]

\[ KE_o = 211750\text{-J} \]
\[ KE_f = 847000\text{-J} \]

\[ KE_o = 2.12 \times 10^5\text{-J} \]
\[ KE_f = 8.47 \times 10^5\text{-J} \]
Kinetic Energy and Work

• In the previous problem the initial energy was 2.12 \times 10^5\text{-J} and the final energy was 8.47 \times 10^5\text{-J} how is this possible if energy must be conserved?

• As there was no stored energy remember that \( W=\Delta KE \).

• The engine does work to the system and increases the cars speed (6.35 \times 10^5\text{-J}).
Types of Stored Energy

• Potential (Gravitational): If a boulder is sitting on the top of a hill, it has the ability to roll down to the bottom (because of gravity).
  – At the top the boulder has a full hill of stored energy.
  – Half way down it only has half a hill of energy.
  – At the bottom it has no more energy from the hill.
  – PE (U) = mgh ([s] Joule)

• Potential (Elastic): If a spring is compressed then it has the ability to release energy. (PE$_e$ = 1/2kx$^2$).
The Flow of Energy

• When looking at closed systems normally there is a flow between Kinetic Energy and Potential Energy. Look at the following:

  - PE = x
  - KE = 0

  - PE = 0.5x
  - KE = 0.5x

  - PE = 0
  - KE = x

• In all cases the total energy is conserved.
Another Exchange

• Here is another look with a graph.

\[ \text{PE} = x \]
\[ \text{KE} = 0 \]
\[ \text{PE} = 0 \]
\[ \text{KE} = x \]
**KE-PE Example I**

- A large chunk of ice with mass 15.00-kg falls from a roof 8.00-m above the ground.
  - What is the KE of the ice before it hits the ground?
  - What is the speed of the ice before it hits the ground?

\[ \Sigma E = KE + PE \rightarrow KE_o + PE_{go} = KE_f + PE_{gf} \]

\[ 0-J + 15.0-\text{kg} \times 9.8-\text{m/s}^2 \times 8.00-\text{m} = KE_f + 0-J \]

\[ KE_f = 1176-J \text{ or } 1.18 \text{ kJ} \]

\[ KE_f = \frac{1}{2} m \times v^2 \]

\[ 1176-J = \frac{1}{2} \times 15.0-\text{kg} \times v^2 \]

\[ v^2 = 156.8-\text{m}^2/\text{s}^2 \]

\[ v = 12.522-\text{m/s} \rightarrow 12.52-\text{m/s} \]
KE-PE Example II

• A ball slams against the ground at 20.00-m/s. Assuming perfect elasticity how high will it bounce?
  – Note even without mass it is possible to solve.
  – A new way to look at this $v_{\text{max bottom}} = h_{\text{max top}}$.
  KE$_{\text{bottom}}$ = PE$_{\text{top}}$ or $\frac{1}{2} mv^2 = mgh$
  $\frac{1}{2} v^2 = gh$
  $\frac{1}{2} (20.00-\text{m/s})^2 = 9.80-\text{m/s}^2 \times h$
  $h = 20.4082-\text{m} \rightarrow 20.41-\text{m}$

This also shows all masses fall at the same rate.
On a frictionless road two cars ($m_1=575.00\text{-kg}$, $m_2=1575.00\text{-kg}$) hit each other and stick together ($v_1=15.00\text{-m/s}$, $v_2=5.00\text{-m/s}$).

- What is the total Energy in the system?
- What is the final velocity of the locked cars?
- What is the final KE of the cars?
Collisions

• \( m_1 = 575.00 \text{-kg} \quad v_1 = 15.00 \text{-m/s} \)
• \( m_2 = 1575.00 \text{-kg} \quad v_2 = 5.00 \text{-m/s} \)
  – What is the total Energy in the system?

\[
E = KE_1 + KE_2 \\
E = \frac{1}{2} 575\text{-kg} \ (15.00\text{-m/s})^2 + \frac{1}{2} 1575\text{-kg} \ (5.00\text{-m/s})^2 \\
E = 84375\text{-J} \rightarrow 84.4\text{-kJ or } 8.44 \times 10^4\text{-J}
\]
Collisions

- $m_1 = 575.00$-kg, $v_1 = 15.00$-m/s, $E = 84.40$-kJ
- $m_2 = 1575.00$-kg, $v_2 = 5.00$-m/s
  - What is the final velocity of the locked cars?

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

$$575$-kg $\times$ $15.00$-m/s $+$ $1575$-kg $\times$ $5$-m/s $=$ $2150$-kg $\times$ $v_f$$$

$$V_f = 7.67442$-m/s $\rightarrow$ $7.67$-m/s
Collisions

- \( m_1 = 575.00 \text{-kg} \quad v_1 = 15.00 \text{-m/s} \quad E = 84.40 \text{-kJ} \)
- \( m_2 = 1575.00 \text{-kg} \quad v_2 = 5.00 \text{-m/s} \quad v_f = 7.67 \text{-m/s} \)
  - What is the final KE of the cars?
  
  \[
  KE = \frac{1}{2} (m_1 + m_2) \times v_f^2 \\
  KE = \frac{1}{2} (2150 \text{-kg}) \times (7.67 \text{-m/s})^2 \\
  KE = 63314\text{-J} \rightarrow 63.3\text{-kJ or } 6.33 \times 10^4\text{-J}
  
  Note in collisions the system is not closed so some energy will be lost